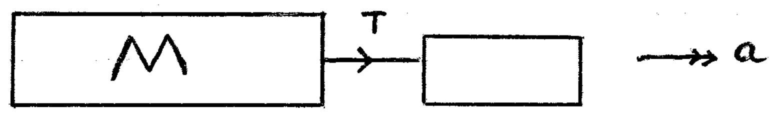
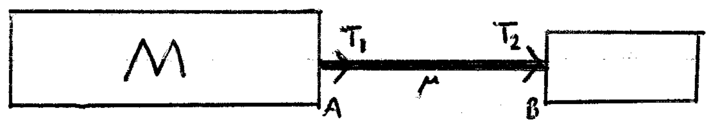
Mechanics Examples Sheet 5 – Solutions

1.



using N2, we have for the caravan

$$\underline{T = Ma}$$

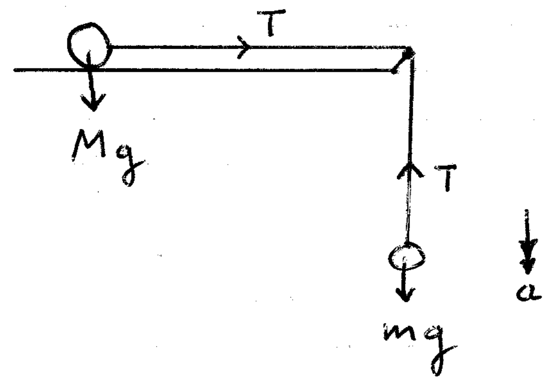


At A: $\underline{T_1 = Ma}$

At B: $\underline{T_2 = (M + \mu)a}$

2.

Because the pulley is smooth the tension in the string will be the same throughout its length



For mass M, we have

$$T = Ma. \qquad -(1)$$

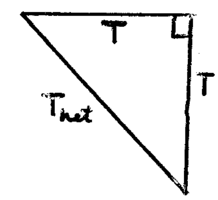
For mass m, the resultant force is

$$mg - T = ma \qquad -(2)$$

On combining (1) and (2), we have

$$\underline{a = \left(\frac{m}{M+m}\right)g}, \quad \underline{T = \left(\frac{Mm}{M+m}\right)g}$$

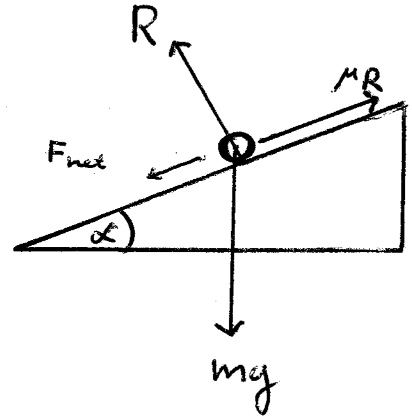
As the tensions in the two sections of the string are perpendicular to each other, the net force, T_{net}, is



$$T_{net}^2 = T^2 + T^2 \Rightarrow T_{net} = \underline{\sqrt{2}\left(\frac{Mm}{M+m}\right)g}$$

3. $\perp$ to plane:

$$R = mg\cos\alpha$$



$\uparrow\uparrow$ to plane:

$$F_{net} = mg\sin\alpha - \mu R$$

$$\Rightarrow \quad ma = mg\sin\alpha - \mu(mg\cos\alpha)$$

$$\Rightarrow \quad \underline{a = g(\sin\alpha - \mu\cos\alpha)}$$

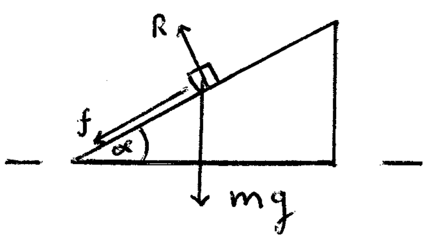
The only differance to the above calculation for the particle being projected up the plane is the reversal of F_{net} and μR. This leads to

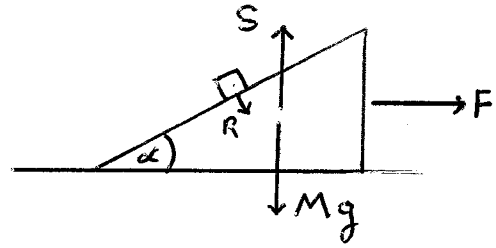
$$F_{net} = mg\sin\alpha + \mu R.$$

Hence,

$$\underline{a = g(\sin\alpha + \mu\cos\alpha)}$$

4.





From N3, when the particle exerts a force on the wedge, the wedge exerts an equal and opposite force on the particle.
The net acceleration of the particle is $f\cos\alpha - F$. So the horizontal forces, from N2, are

$$\underline{R\sin\alpha = m(f\cos\alpha - F)} \qquad -(2)$$

The vertical forces on the particle are

$$\underline{R\cos\alpha = m(g - f\sin\alpha)} \qquad -(3)$$

Horizontal forces on wedge: $\underline{R\sin\alpha = MF}$ $\qquad -(4)$

Vertical forces on wedge: $\underline{R\cos\alpha = S - Mg}$ $\qquad -(1)$

Substituting (4) into (2) & (3):

$MF\sin\alpha = mf\cos\alpha\sin\alpha - mF\sin\alpha$, $\underline{\dfrac{MF\cos^2\alpha}{\sin\alpha}} = mg\cos\alpha - mf\cos\alpha\sin\alpha$

On eliminating $mf\cos\alpha\sin\alpha$:

$MF\sin^2\alpha + mF\sin^2\alpha = mg\cos\alpha\sin\alpha - MF\cos^2\alpha \Rightarrow$ <u>result</u>